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Transport of energy in polychromatic fluid gravity waves

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Abstract Potential theory is used to derive closed-form expressions for the energy flux in propagating polychromatic surface gravity waves for water of infinite depth. The energy flux vector has been obtained as convolution-type integrals of the wave-elevation time series at one point on the surface. An expression for the integrated flux through a vertical surface is also given. It is shown that the integrated flux cannot be negative at any instant in time.

Keywords Energy transport · Polychromatic waves · Potential theory · Water waves · Wave energy

1 Introduction

Ocean waves carry considerable amounts of energy with them as they propagate. Propagating over large distances from continent to continent, the ocean waves are almost loss-free. They also reveal some very interesting properties as a source of renewable energy such as relatively high average power density, compared with sun and wind. The commercial conversion of the hydrodynamic energy in waves into more useful forms, such as electric energy, poses a considerable engineering challenge. For a survey see [1]. There are numerous suggestions in the literature on how to convert wave energy into electricity; see for example [2]. A few of these are currently being tested in different parts of the world [3, 4]. To be able to design optimized wave-energy converters, it is necessary to describe and understand exactly how energy is transported in waves.

The objective of this paper is to describe the instantaneous energy flux in waves from time series of wave elevation. For monochromatic waves or waves with a narrow spectral distribution, the energy transport has been studied in a variety of different cases, see e.g., [5, Sects. 174 and 230, 6, 7, Sect. 3.3]. Recently Tulin [8] treated the case of monochromatic gravity and surface-tension waves in water of arbitrary depth. In the present paper we make no assumptions on the spectral distribution of the waves but instead limit the scope to potential gravity waves in two dimensions at infinite depth and use linearised free-surface boundary conditions. In Sect. 2 we derive an expression with which the instantaneous energy-flux vector can be calculated from a wave-elevation time series. We also derive an expression for the total instantaneous energy flux through a vertical plane perpendicular to the direction of propagation of the wave. This is presented in Sect. 3 where we also prove that this total energy flux cannot be

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negative during any instant of time for a propagating wave. To our knowledge, such closed-form expressions of the instantaneous energy flux in polychromatic waves have not yet been published.

2 Energy flux in linear gravity waves

Water waves with amplitudes that are small compared with wave length can, with good accuracy, be described by linear potential theory; see e.g. [7, Chap. 3–5] or [9, Chap. 4]. This assumes an incompressible, non-viscous and rotation-free fluid. With these assumptions the fluid motion can be expressed through a fluid velocity potential ϕ satisfying the Laplace equation $\nabla^2\phi = 0$. The fluid velocity is expressed as the gradient of the velocity potential. For small amplitudes the boundary conditions can be linearized and the free surface can be replaced with the undisturbed fluid surface ($z = 0$) where the potential satisfies the following two boundary conditions:

(i) The linearized Dynamic Free Surface Boundary Condition (DFSBC):

$$g\eta + \left.\frac{\partial\phi}{\partial t}\right|_{z=0} = 0, \tag{1}$$

where η is the surface elevation, ϕ is velocity potential, and g is the acceleration of gravity, and

(ii) The linearized Kinematic Free Surface Boundary Condition (KFSBC):

$$\left.\frac{\partial\phi}{\partial z}\right|_{z=0} = \frac{\partial\eta}{\partial t}. \tag{2}$$

The Bottom Boundary Condition (BBC) specifies that the fluid velocity must be parallel to the bottom surface. For water of infinite depth the BBC can be replaced with the condition that $\nabla\phi \to 0$. The boundary conditions are shown in Fig. 1.

The Laplace equation for the potential together with the boundary conditions give harmonic solutions for the velocity potential:

$$\phi_{\omega,k}(x, z, t) = \Re e\left(\mathrm{e}^{|k|z}\frac{Hg}{2\omega}\mathrm{e}^{\mathrm{i}(kx-\omega t)}\right), \tag{3}$$

where H is the peak-to-peak height of the harmonic wave and ω its angular frequency. The wavenumber k and the angular frequency ω are related by the deep-water dispersion relation

$$\omega^2 = \pm gk. \tag{4}$$

Equation 3 describes waves propagating in the direction of the positive x-axis if ω and k have the same sign. An arbitrary wave propagating in the positive x-direction can be written as a superposition of solutions of the form (3) where ω and k have the same sign:

$$\phi(x, z, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{\phi}(\omega)\mathrm{e}^{|k|z}\mathrm{e}^{\mathrm{i}(kx-\omega t)}\mathrm{d}\omega, \tag{5}$$

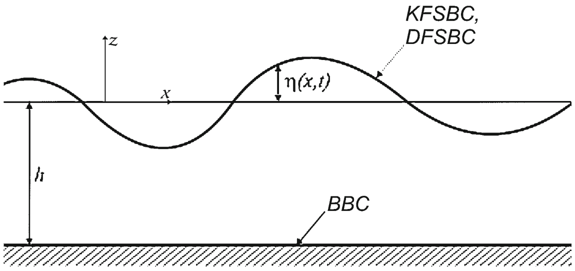

Fig. 1 Definition of parameters for the fluid domain

where $\hat{\phi}(\omega)$ is the Fourier transform of $\phi(0, 0, t)$.

$$\hat{\phi}(\omega) = \int_{-\infty}^{\infty} \phi(0, 0, t) e^{-i\omega t} dt \tag{6}$$

Changing the integration order gives:

$$\phi(x, z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi(0, 0, t') \left(\int_{-\infty}^{\infty} e^{|k|z} e^{ikx} e^{i\omega(t'-t)} d\omega \right) dt'. \tag{7}$$

which is valid for $z < 0$. The energy transport per unit time and unit area for surface waves equals the pressure times the fluid velocity $p_{\text{tot}}\vec{v} = p_{\text{stat}}\vec{v} + p\vec{v}$. Here p_{tot} is the total pressure, p_{stat} the static pressure, p the dynamic pressure and $\vec{v}$ the fluid velocity. However, only the work done by the pressure fluctuations $p\vec{v}$ is associated with utilizable work. The work done by the static background is of no consequence in this context and will not be considered in the following. This part also averages to zero. The energy-flux vector can therefore be defined as:

$$\vec{j} = (j_x, j_y, j_z) = (pv_x, pv_y, pv_z) = \rho \frac{\partial \phi}{\partial t} \left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z} \right), \tag{8}$$

where ρ is the density of the fluid. For a plane parallel wave propagating in the positive x-direction the y-component of $\vec{j}$ vanishes, as there is no motion in the y-direction. It turns out that the energy-flux vector in the plane $x = 0$ can be expressed as certain integrals over time, quadratic in the wave elevation $\eta(t)$. To find such expressions we wish to express the time and spatial derivatives of the potential in (8) as integrals of the wave elevation separately.

By taking the time derivative of (7), setting $x = 0$, shifting the differentiation from t to t', integrating partially with respect to t' and finally by using the expression for linearized DFSBC (1), $\eta = -\partial\phi/g\partial t|_{z=0}$, one finds:

$$\frac{\partial \phi}{\partial t}(0, z, t) = \frac{g}{\pi} \int_{-\infty}^{\infty} \eta(t') \int_{0}^{\infty} e^{|k|z} \cos(\omega(t' - t))\, d\omega\, dt' \tag{9}$$

for $z < 0$. The inner integral can be found in standard integral tables, and one obtains the result that the time derivative can be expressed as a convolution of the wave elevation with a Gauss kernel:

$$\frac{\partial \phi}{\partial t}(0, z, t) = -g \int_{-\infty}^{\infty} K(-z/g, t' - t)\eta(t')\, dt' \quad \text{with} \quad K(\zeta, \tau) = \frac{1}{2\sqrt{\pi\zeta}} e^{-\frac{\tau^2}{4\zeta}}, \tag{10}$$

which is also valid for $z < 0$. It is easy to check that in the limit $z \to 0^-$ the r.h.s of (10) approaches $-g\eta$, which is in agreement with the DFSBC. The spatial derivative of the potential with respect to the depth can be obtained in a similar manner. Differentiating (7) w.r.t. z and setting x to zero yields:

$$\begin{aligned} \frac{\partial \phi}{\partial z}(0, z, t) &= \frac{1}{\pi} \int_{-\infty}^{\infty} \phi(0, 0, t') \int_{0}^{\infty} |k|\, e^{|k|z} \cos(\omega(t' - t))\, d\omega\, dt' \\ &= \frac{1}{g\pi} \int_{-\infty}^{\infty} \phi(0, 0, t') \int_{0}^{\infty} \omega^2 e^{|k|z} \cos(\omega(t' - t))\, d\omega\, dt'. \end{aligned} \tag{11}$$

Partial integration with respect to t' and then making use of the linearized DFSBC again gives:

$$\frac{\partial \phi}{\partial z}(0, z, t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \eta(t') \int_{0}^{\infty} \omega e^{\omega^2 z/g} \sin(\omega(t' - t))\, d\omega\, dt'. \tag{12}$$

The inner integral can be calculated explicitly [10, Eq. 3.952.1] as:

$$\frac{\partial \phi}{\partial z}(0, z, t) = \int_{-\infty}^{\infty} L(-z/g, t' - t)\eta(t')\, dt' \quad \text{with} \quad L(\zeta, \tau) = \frac{\tau}{4\sqrt{\pi}\zeta^{3/2}} e^{-\frac{\tau^2}{4\zeta}}, \tag{13}$$

valid for $z < 0$. In the limit $z \to 0^-$ the r.h.s of (13) approaches $\partial\eta/\partial t$, which is in agreement with the KFSBC. Finally we also need an expression for the derivative of ϕ with respect to x. This case turns out to be somewhat more complicated due to the differentiation in the direction of propagation, but it can be solved explicitly nevertheless. Differentiating (7) w.r.t. x and then setting x to zero gives, for $z < 0$:

$$\frac{\partial\phi}{\partial x}(0,z,t) = -\frac{1}{\pi}\int_{-\infty}^{\infty}\phi(t')\int_{0}^{\infty}\frac{\omega^2}{g}\mathrm{e}^{\frac{z\omega^2}{g}}\sin(\omega(t'-t))\,\mathrm{d}\omega\,\mathrm{d}t' = \frac{1}{\pi}\int_{-\infty}^{\infty}\eta(t')\int_{0}^{\infty}\mathrm{e}^{\frac{z\omega^2}{g}}\omega\cos(\omega(t'-t))\,\mathrm{d}\omega\,\mathrm{d}t', \quad (14)$$

where the last equality follows from partial integration and inserting the linearized DFSBC. This integral can also be computed explicitly and the result is:

$$\frac{\partial\phi}{\partial x}(0,z,t) = \int_{-\infty}^{\infty} M(-z/g, t'-t)\eta(t')\,\mathrm{d}t', \quad (15)$$

where the integration kernel M is given by

$$M(\zeta,\tau) = \frac{1}{2\pi\zeta} - \frac{|\tau|}{2\pi\zeta^{3/2}}\mathrm{e}^{-\frac{\tau^2}{4\zeta}}\int_{0}^{\frac{|\tau|}{2\sqrt{\zeta}}}\mathrm{e}^{x^2}\mathrm{d}x. \quad (16)$$

The last term contains Dawson's integral, which is closely related to the error function, for which there are efficient numerical computation algorithms [11, p. 105]. The limit $z \to 0^-$ of (15) exists and in fact it can be shown from (15) that:

$$\lim_{z\to 0^-}\frac{\partial\phi}{\partial x}(0,z,t) = \frac{1}{\pi}\int_{-\infty}^{\infty}(\eta'(t-t') - \eta'(t))\frac{\mathrm{d}t'}{t'}, \quad (17)$$

(where $\eta'(t) \equiv \frac{\partial\eta}{\partial t}$) thereby expressing the horizontal surface velocity as an integral over the vertical surface velocity at the same point. The energy-flux vector is now obtained by inserting expressions (10), (13) and (15) into (8). This gives the instantaneous energy-flux vector $\vec{j}$ at $x = 0$ and depth $-z$, as:

$$\begin{aligned}
&\vec{j} = (j_x, j_y, j_z),\\
&j_x(z,t) = \rho g\int_{-\infty}^{\infty}K(-z/g, t'-t)\eta(t')\mathrm{d}t' \times \int_{-\infty}^{\infty}M(-z/g, t'-t)\eta(t')\mathrm{d}t',\\
&j_y(z,t) = 0,\\
&j_z(z,t) = \rho g\int_{-\infty}^{\infty}K(-z/g, t'-t)\eta(t')\mathrm{d}t' \times \int_{-\infty}^{\infty}L(-z/g, t'-t)\eta(t')\mathrm{d}t',
\end{aligned} \quad (18)$$

where the integration kernels K, L and M are given in (10), (13) and (16), respectively. The energy-flux vector for a typical wave is plotted in Fig. 4. The wave-elevation data for this sample wave is shown in Fig. 2a. The data used for this plot were measured outside Lysekil off the west coast of Sweden and were selected to represent typical ocean waves at this location. For the measurement we used a commercial system: Omni directional Wave Raider from Datawell. The data were obtained at a sampling rate of 2.56 Hz. As Figs. 3 and 4 illustrate, the wave-energy transport varies rapidly with time. For shorter periods, for example between 40 and 50s in Fig. 2a, the wave elevation is moderate, resulting in an energy transport which is very small compared to higher wave elevations. During one wave period the energy transport fluctuates in time, from peaks of 70 kW/m to 0 kW/m in just a few seconds. The energy transport decreases very rapidly with increasing distance from the surface, which is illustrated in Fig. 2c. Just a few meters under the surface the energy flux has decreased considerably.

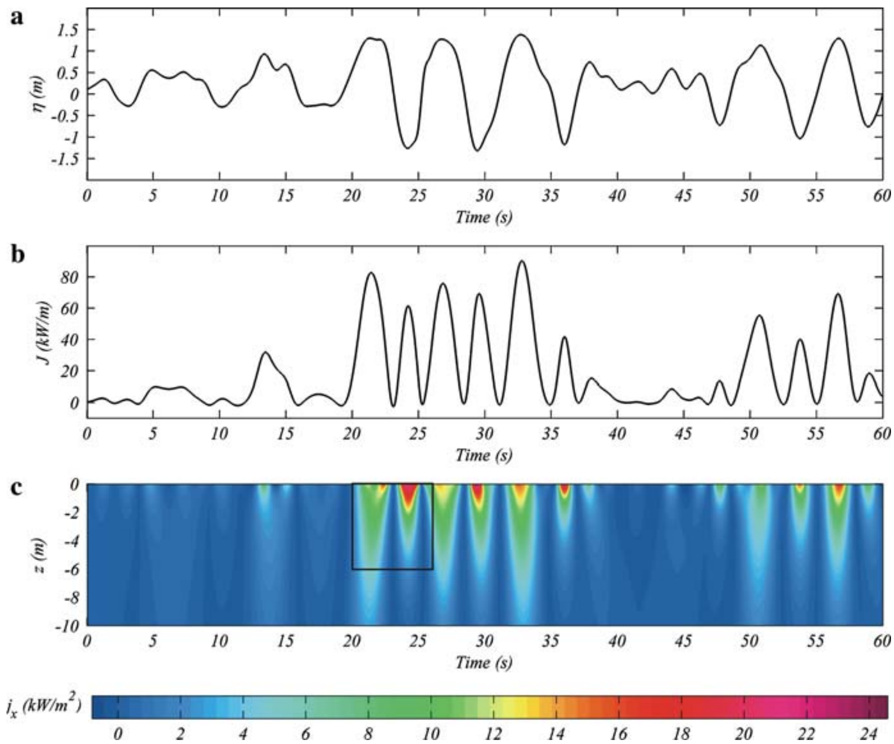

Fig. 2 (**a**) Wave elevation for a sample wave (measured data), (**b**) total energy flux integrated over a vertical plane perpendicular to the wave direction of propagation, (**c**) horizontal component of the energy-flux vector

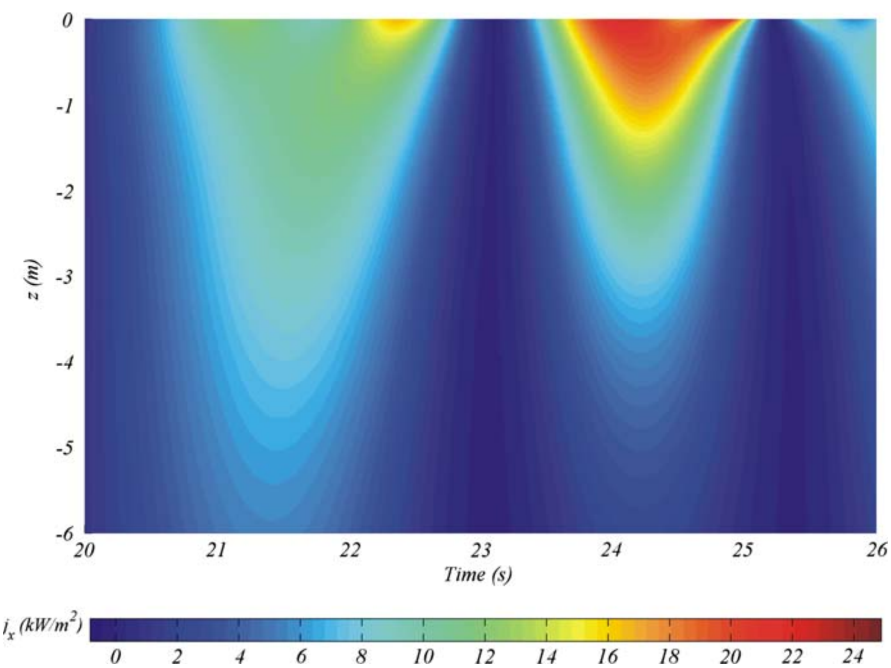

Fig. 3 Total energy flux through a vertical plane—detail from Fig. 2c

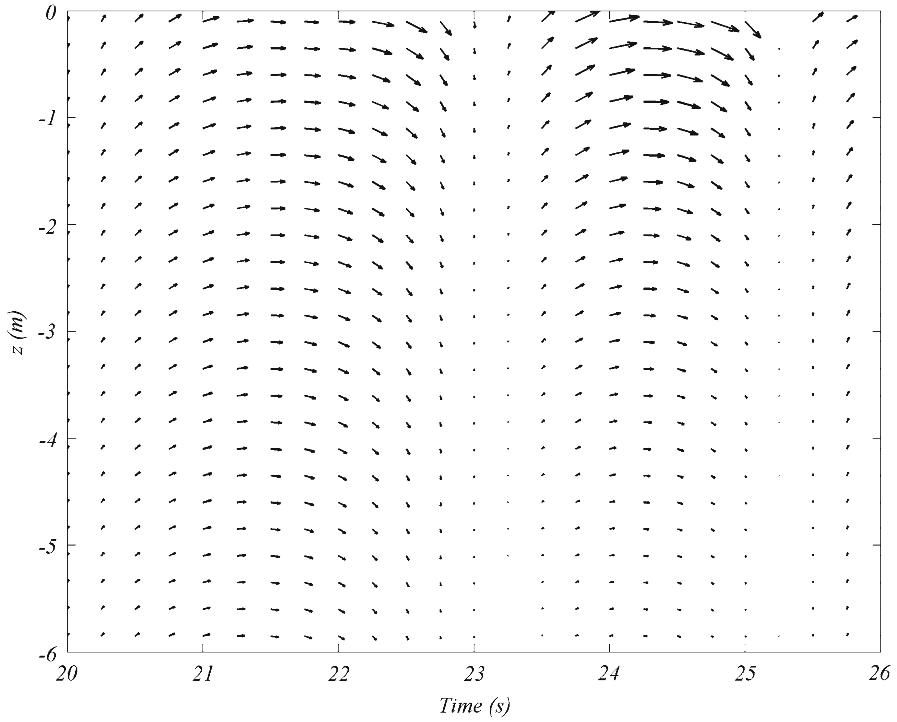

Fig. 4 The energy-flux vector as a function of time and depth for the wave, described in Fig. 2a

3 Total energy flux

The work that the volume of the water to the left of the origin ($x < 0$) exerts on the volume to the right ($x > 0$) is equal to the total instantaneous energy flux through the surface $x = 0$. The total energy flux through any surface S equals the surface integral $\int_S \vec{j} \cdot \vec{n} \mathrm{d}S$ where $\vec{n}$ is the outward unit normal vector to S. For the surface given by $x = 0$ this vector is the constant unit vector in the positive x-direction and therefore the total instantaneous energy flux per meter of wave front is $J(t) = \int_{-\infty}^{0} j_x(z, t)\mathrm{d}z$. The time average $\bar{J}$ of $J(t)$, i.e., the average energy flux per meter wave front, is given by the well-known expression:

$$\bar{J} = kTH^2, \tag{19}$$

where $k = \rho g^2/(32\pi)$, T is the energy period, and H is the significant wave elevation. For plane waves with wavefronts not parallel with the surface $x = 0$ the integrated instantaneous flux is constant in time and equals $\bar{J}$ times the cosine of the incidence angle. This follows since the y-integration then in effect averages over the wave. However, for a parallel wave, this is no longer the case and the integrated flux varies in time. An expression for the instantaneous energy flux $J(t)$ can (with considerable effort) be derived by integrating $j_x(z, t)$ as given in (18) with respect to z, but there is a somewhat simpler route. By inserting (9) and (14) into $j_x = -\rho \frac{\partial\phi}{\partial t}\frac{\partial\phi}{\partial x}$ instead and integrating with respect to z, we obtain:

$$J(t) = \frac{\rho g^2}{\pi^2} \int_0^{\infty} \int_{-\infty}^{\infty} \eta(t') \int_0^{\infty} \exp\left(\frac{z\omega'^2}{g}\right) \cos(\omega'(t'-t))\mathrm{d}\omega'\mathrm{d}t' \times \int_{-\infty}^{\infty} \eta(t'') \int_0^{\infty} \exp\left(\frac{z\omega''^2}{g}\right) \omega'' \cos(\omega''(t''-t))\mathrm{d}\omega''\mathrm{d}t''\mathrm{d}z. \tag{20}$$

Changing the integration order, performing the z-integration first, yields:

$$J(t) = \frac{\rho g^2}{\pi^2} \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \eta(t')\eta(t'') \int\limits_{0}^{\infty} \int\limits_{0}^{\infty} \frac{\omega''}{\omega'^2 + \omega''^2} \cos(\omega'(t'-t)) \cos(\omega''(t'-t)) \mathrm{d}\omega' \mathrm{d}\omega'' \mathrm{d}t' \mathrm{d}t''. \tag{21}$$

The integrations over ω' and ω'' can be performed explicitly:

$$\begin{aligned} \int\limits_{0}^{\infty} \int\limits_{0}^{\infty} \frac{\omega''}{\omega'^2 + \omega''^2} \cos(\omega'(t'-t)) \cos(\omega''(t'-t)) \mathrm{d}\omega' \mathrm{d}\omega'' &= \frac{\pi}{2} \int\limits_{0}^{\infty} \exp\left(-\left|t - t'\right| \omega''\right) \cos(\omega''(t'-t)) \mathrm{d}\omega'' \\ &= \frac{\pi}{2} \frac{\left|t' - t\right|}{(t'-t)^2 + (t''-t)^2}. \end{aligned} \tag{22}$$

Inserting this kernel into (21) and symmetrizing the kernel and shifting the integration variables, we obtain the desired expression for the instantaneous total energy flux as follows:

$$J(t) = \frac{\rho g^2}{2\pi} \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} I(t', t'')\eta(t'-t)\eta(t''-t) \mathrm{d}t' \mathrm{d}t'' \quad \text{with} \quad I(t', t'') = \frac{\left|t'\right| + \left|t''\right|}{t'^2 + t''^2}. \tag{23}$$

Owing to its relative simplicity, we have found this expression to be very useful for efficient computation of the total instantaneous energy flux in waves. Plots of the energy flux for a sample wave is shown in Figs. 2–4.

From Fig. 2b it can be observed that the total instantaneous energy flux for a typical wave always seems to be positive. However, it is not in any way obvious from (23) that this should be the case in general. Below we show that the flux is indeed always positive or zero, but never negative, or expressed differently: that the kernel I in (23) is positive semi-definite. For technical reasons we will assume that $\eta \in \mathcal{L}^2(\mathbb{R})$, i.e., that η is square-integrable over the interval $(-\infty, \infty)$. This is clearly no limitation for the physical problem as we can always limit ourselves to waves of finite, but arbitrarily long, duration.

Since the origin for the time coordinate is arbitrary, we lose no generality in the argument by setting $t = 0$ in (23):

$$J(t=0) = \frac{\rho g^2}{2\pi} \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \frac{\left|t'\right| + \left|t''\right|}{t'^2 + t''^2} \eta(t')\eta(t'') \, \mathrm{d}t' \, \mathrm{d}t''. \tag{24}$$

This can be further simplified by splitting η into even and odd parts. The energy flux J then only depends on the even part η_e:

$$J(t=0) = \frac{2\rho g^2}{\pi} \int\limits_{0}^{\infty} \int\limits_{0}^{\infty} \frac{t' + t''}{t'^2 + t''^2} \eta_e(t')\eta_e(t'') \, \mathrm{d}t' \, \mathrm{d}t''. \tag{25}$$

Substituting $t' = \exp(x)$, $t'' = \exp(y)$ and introducing a new function $\xi(x) = \mathrm{e}^{x/2}\eta_e(\mathrm{e}^x)$, Eq. 25 can be written in a form that contains a convolution integral:

$$J(t=0) = \frac{2\rho g^2}{2\pi} \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \frac{\cosh\left(\frac{x-y}{2}\right)}{\cosh(x-y)} \xi(x)\xi(y) \, \mathrm{d}x \, \mathrm{d}y = \frac{2\rho g^2}{\pi} (\xi, L * \xi), \tag{26}$$

where $(f, g) = \int_{-\infty}^{\infty} \overline{f(x)} g(x) \, \mathrm{d}x$ denotes the inner product in $\mathcal{L}^2(\mathbb{R})$, $*$ denotes the convolution:

$$(f * g)(x) = \int\limits_{-\infty}^{\infty} f(x-y)g(y) \, \mathrm{d}y \quad \text{and} \quad L(x) = \frac{\cosh(x/2)}{\cosh(x)}. \tag{27}$$

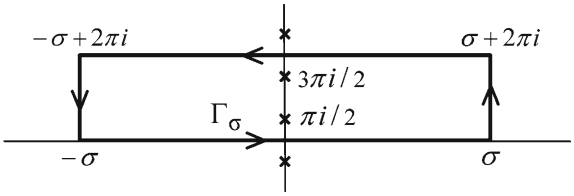

Fig. 5 Integration path in Eq. 31

Note that $L \in \mathcal{L}^2(\mathbb{R})$, and that $\xi \in \mathcal{L}^2(\mathbb{R})$ if and only if $\eta_e \in \mathcal{L}^2(\mathbb{R})$ since:

$$\int_0^\infty |\eta_e(t')|^2 \,\mathrm{d}t' = \int_{-\infty}^\infty |\eta_e(e^x)|^2 \,\mathrm{e}^x \mathrm{d}x = \int_{-\infty}^\infty |\xi(x)|^2 \,\mathrm{d}x. \tag{28}$$

Using Parseval's identity and the convolution theorem for Fourier integrals, we get:

$$(\xi, L * \xi) = \frac{1}{2\pi}(\hat{\xi}, \hat{L}\hat{\xi}). \tag{29}$$

The Fourier integral of L:

$$\hat{L}(\omega) = \int_{-\infty}^\infty L(x)\exp(-\mathrm{i}\omega x)\mathrm{d}x \tag{30}$$

can be calculated explicitly by choosing an integration path in the complex plane as in Fig. 5:

$$\lim_{\sigma\to\infty}\int_{\Gamma_\sigma} L(x)\exp(-\mathrm{i}\omega x)\mathrm{d}x = (1+\exp(2\pi\omega))\hat{L}(\omega) = 2\pi\mathrm{i}\{\mathrm{Res}(\pi\mathrm{i}/2) + \mathrm{Res}(3\pi\mathrm{i}/2)\}. \tag{31}$$

Here we add the residues from the two poles $\pi\mathrm{i}/2$ and $3\pi\mathrm{i}/2$ of the integrand $L(x)\exp(-\mathrm{i}\omega x)$ inside the counter Γ_σ.

By computing the residues in (31) and solving for $\hat{L}$ we obtain the expression

$$\hat{L}(\omega) = \pi\sqrt{2}\frac{\exp(\pi\omega/2)+\exp(3\pi\omega/2)}{1+\exp(2\pi\omega)} = \pi\sqrt{2}\frac{\cosh(\pi\omega/2)}{\cosh(\pi\omega)} = \pi\sqrt{2}L(\pi\omega), \tag{32}$$

which clearly is always positive. The energy flux J is therefore also positive unless $\hat{\xi}$ vanishes almost everywhere.

$$J(t=0) = \frac{\rho g^2}{\pi^2}(\hat{\xi}, \hat{L}\hat{\xi}) = \sqrt{2}\frac{\rho g^2}{\pi}\int_{-\infty}^\infty \left|\hat{\xi}(\omega)\right|^2 \frac{\cosh(\pi\omega/2)}{\cosh(\pi\omega)}\mathrm{d}x \ge 0. \tag{33}$$

The variable $\hat{\xi}$ only vanishes almost everywhere if η is odd. Therefore the kernel I in (23) is positive semi-definite, or expressed differently, the instantaneous total energy flux is never negative.

4 Conclusion

To be able to design efficient wave-energy converters, it is necessary to understand how and where energy is transported in waves. In this paper we have derived closed-form expressions for the energy flux in plane-parallel propagating surface gravity waves of arbitrary shape by using potential theory. The method consists of the expansion of arbitrary waves into harmonic fluid velocity-potential solutions of the Laplace equation and the boundary conditions. By this method the energy-flux vector has been obtained as convolution-type integrals of the wave elevation time series at one point. These integrals contain Gauss-type kernels and a kernel with transcendental functions. Numerical algorithms to perform these calculations effectively exist. It is therefore possible to calculate the power flux almost in real-time, something that we hope will prove useful for optimising the power conversion in wave-power conversion units. We have also derived a simple expression for the total instantaneous energy-flux crossing a plane perpendicular to the direction of propagation. It was shown that this total flux can be written as a double integral, quadratic in the wave elevation, with a rational kernel. This kernel was shown to be positive semi-definite, showing that the total flux cannot be negative for a propagating wave, irrespective of the shape of the waves.

As an example the power flux was calculated using a wave-elevation data measured off the Swedish west coast. As illustrated in Figs. 2–4 the wave-energy transport varies rapidly and it can also be noted that the energy transport decreases very rapidly as well with increasing distance from the surface. For wave-power utilization based on point absorbers this will lead to a strongly varying available power which in turn gives a fluctuating output power. This poses quite a challenge in designing this type of wave-power plant. On the other hand the rapid decrease of power flux with dept is very advantageous for proposed wave-power concepts using a point absorber with a seafloor-mounted generator. The absorber position is where all the energy is transported and the generator is almost unaffected by the flow.

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